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NEW CONCEPTS OF ISOMORPHISM ON NEUTROSOPHIC FUZZY GRAPHS

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Abstract: In this article, a new dimension named isomorphism, weak and co-weak isomorphism in the neighbourly (highly) streak erratic neutrosophic fuzzy graphs and its complement are introduced and some of this properties are discussed. Moreover, isomorphic properties in the $\mu-$ complement of neighbourly (highly) streak erratic neutrosophic fuzzy graph are established.

Keywords and Phrases: Neutrosophic fuzzy graphs, neighbourly streak erratic neutrosophic fuzzy graph, highly streak erratic neutrosophic fuzzy graph.

2020 Mathematics Subject Classification: 05C12, 05C72.

1. Introduction

Smarandache [8] proposed Neutrosophic Graphs in some special types such as neutrosophic multipolar graphs. Broumi [1] introduced certain types of single valued neutrosophic graphs. Neighbourhood degree of a vertex in neutrosophic graphs were introduced in [2]. Up to now, to the best of our knowledge there has been no study on isomorphic concepts of neutrosopic fuzzy graph. This we have proposed in this paper, a new concept of isomorphism on neutrosophic fuzzy graph. We have provided some examples when and where necessary. Also we have proved some results.

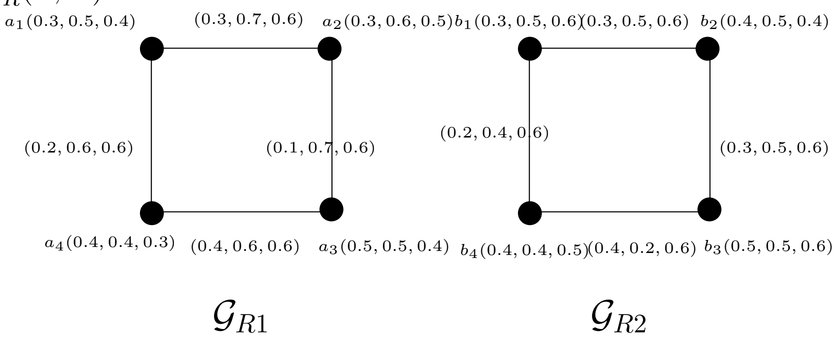
2. Isomorphism on Streak Erratic Neutrosophic Fuzzy Graphs

Definition 2.1. *A homomorphism l of neighbourly streak erratic neutrosophic fuzzy graph(highly streak erratic neutrosophic fuzzy graph) $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ is a map $l\colon V_1 \to V_2$ that satisfies the following condition*

(i) $\mathcal{T}_N(m) \leq \mathcal{T}_N(l(m))$, $\mathcal{I}_N(m) \geq \mathcal{I}_N((m))$ *and* $\mathcal{F}_N(m) \geq \mathcal{F}_N(l(m))$

(ii) $\mathcal{T}_M(mk) \leq \mathcal{T}_M(l(mk))$, $\mathcal{I}_M(mk) \geq \mathcal{I}_M((mk))$ *and* $\mathcal{F}_M(mk) \geq \mathcal{F}_M(l(mk))$.

Example 2.2. Consider neighbourly streak erratic neutrosophic fuzzy graph on $H_R^*(V,E)$.



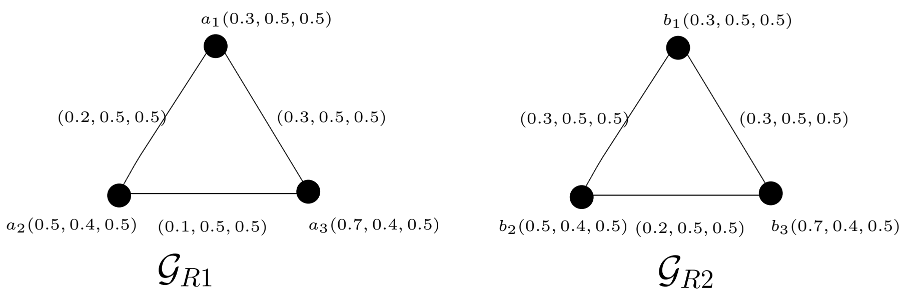
There is a homomorphism $l\colon V_1 \to V_2$ such that $l(a_1) = b_1$, $l(a_2) = b_2$, $l(a_3) = b_3, l(a_4) = b_4$.

Definition 2.3. *A weak isomorphism l of neighbourly streak erratic neutrosophic fuzzy graph(highly streak erratic neutrosophic fuzzy graph) $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ is a 1-1 and onto map where $l : V_1 \to V_2$ which satisfies the following conditions.*

(i) l is homomorphism

(ii) $\mathcal{T}_N(m) = \mathcal{T}_N(l(m))$, $\mathcal{I}_N(m) = \mathcal{I}_N((m))$ *and* $\mathcal{F}_N(m) = \mathcal{F}_N(l(m))$.

Example 2.4. Given neighbourly streak erratic neutrosophic fuzzy graph on $H_R^*(V,E)$.



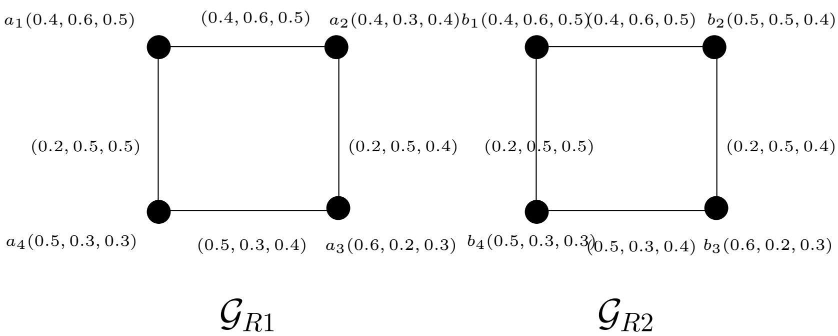
From the example given above we see that there is a weak isomorphism $k\colon V_1 \to V_2$ such that $k(a_1) = b_1$, $k(a_2) = b_2$, $k(a_3) = b_3$.

Definition 2.5. *A co-weak isomorphism of neighbourly streak erratic neutrosophic fuzzy graph(highly streak erratic neutrosophic fuzzy graph) $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ is a 1-1 and onto map $l\colon V_1 \to V_2$ that fulfill the said condition*

(i) l is homomorphism

(ii) $\mathcal{T}_M(mk) = \mathcal{T}_M(l(mk))$, $\mathcal{I}_M(mk) = \mathcal{I}_M((mk))$ and $\mathcal{F}_M(mk) = \mathcal{F}_M(l(mk))$

Example 2.6. Consider highly streak erratic neutrosophic fuzzy graph on $H_R^*(V, E)$.



Definition 2.7. *An isomorphism k of neighbourly streak erratic neutrosophic fuzzy graph(highly streak erratic neutrosophic fuzzy graph) $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ is a 1-1, onto map $l\colon V_1 \to V_2$ that satisfies*

(i) $\mathcal{T}_N(m) = \mathcal{T}_N(l(m))$, $\mathcal{I}_N(m) = \mathcal{I}_N((m))$ and $\mathcal{F}_N(m) = \mathcal{F}_N(l(m))$

(ii) $\mathcal{T}_M(mk) = \mathcal{T}_M(l(mk))$, $\mathcal{I}_M(mk) = \mathcal{I}_M((mk))$ and $\mathcal{F}_M(mk) = \mathcal{F}_M(l(mk))$

Theorem 2.8. *If any two neighbourly streak erratic neutrosophic fuzzy graphs are isomorphic then the order and size of the graph remains same.*

Proof. If $l\colon \mathcal{G}_{R1} \to \mathcal{G}_{R2}$ be an isomorphism between the two neighbourly streak erratic neutrosophic fuzzy graph then,

$\mathcal{T}_N(m) = \mathcal{T}_N(l(m))$, $\mathcal{I}_N(m) = \mathcal{I}_N((m))$ and $\mathcal{F}_N(m) = \mathcal{F}_N(l(m))$

$\mathcal{T}_M(mk) = \mathcal{T}_M(l(mk))$, $\mathcal{I}_M(mk) = \mathcal{I}_M((mk))$ and $\mathcal{F}_M(mk) = \mathcal{F}_M(l(mk))$

$$\begin{aligned} O(\mathcal{G}_{R1}) &= \left(\sum \mathcal{T}_N(v_1), \sum \mathcal{I}_N(v_1), \sum \mathcal{F}_N(v_1)\right) \\ &= \left(\sum \mathcal{T}_N(l(v_1)), \sum \mathcal{I}_N((v_1)), \sum \mathcal{F}_N(l(v_1))\right) \\ &= \left(\sum \mathcal{T}_N(v_2), \sum \mathcal{I}_N(v_2), \sum \mathcal{F}_N(v_2)\right) = O(\mathcal{G}_{R2}) \\ S(\mathcal{G}_{R1}) &= \left(\sum \mathcal{T}_M(v_1w_1), \sum \mathcal{I}_M(v_1w_1), \sum \mathcal{F}_M(v_1w_1)\right) \\ &= \left(\sum \mathcal{T}_M(l(v_1w_1)), \sum \mathcal{I}_M((v_1w_1)), \sum \mathcal{F}_M(l(v_1w_1))\right) \\ &= \left(\sum \mathcal{T}_M(v_2w_2), \sum \mathcal{I}_M(v_2w_2), \sum \mathcal{F}_M(v_2w_2)\right) = S(\mathcal{G}_{R2}). \end{aligned}$$

Remark 2.9. *The Theorem 2.8 mentioned above is true for highly streak erratic* *neutrosophic fuzzy graph.*

Remark 2.10. *The converse of Theorem 2.8 need not be true for both the neighbourly streak erratic neutrosophic fuzzy graph and the highly streak erratic neutrosophic fuzzy graph.*

Theorem 2.11. *If the neighbourly streak erratic neutrosophic fuzzy graphs(highly streak erratic neutrosophic fuzzy graph) are weak isomorphic then the order of the isomorphism is same.*
Proof. It is similar to Theorem 2.8.

Remark 2.12. Neighbourly streak erratic neutrosophic fuzzy graph(highly streak erratic neutrosophic fuzzy graph) of equal order need not be weak isomorphic.

Theorem 2.13. *If neighbourly streak erratic neutrosophic fuzzy graphs(highly streak erratic neutrosophic fuzzy graph)are co-weak isomorphic then its number of streaks will be same.*
Proof. Similar to Theorem 2.8.

Remark 2.14. *Neighbourly streak erratic neutrosophic fuzzy graph (highly streak erratic neutrosophic fuzzy graph) of equal size need not be co-weak isomorphic.*

Theorem 2.15. *If $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are neighbourly streak erratic neutrosophic fuzzy graphs that are isomorphic then the degrees of corresponding vertices u and $k(u)$ are same.*
Proof. Assume that $l\colon \mathcal{G}_{R1} \to \mathcal{G}_{R2}$ is an isomorphism between the two neighbourly streak erratic neutrosophic fuzzy graph then

$\mathcal{T}_M(v_1w_1) = \mathcal{T}_M(l(v_1w_1)), \mathcal{I}_M(v_1w_1) = \mathcal{I}_M((v_1w_1))$ and $\mathcal{F}_M(v_1w_1) = \mathcal{F}_M(l(v_1w_1))$.
$d_T(v_1) = \sum \mathcal{T}_M(v_1w_1) = \sum \mathcal{T}_M(l(v_1w_1)) = d_T(l(v_1))$.
$d_I(v_1) = \sum \mathcal{I}_M(v_1w_1) = \sum \mathcal{I}_M((v_1w_1)) = d_I(l(v_1))$.
$d_F(v_1) = \sum \mathcal{F}_M(v_1w_1) = \sum \mathcal{F}_M(l(v_1w_1)) = d_F(l(v_1))$.

Thus the degrees of corresponding vertices of $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are same.

Theorem 2.16. *Let $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ be two highly streak erratic neutrosophic fuzzy graphs. $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are isomorphic if and only if their complement are also isomorphic. But the complement need not be highly streak erratic.*
Proof. Assume that $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are isomorphic. There exists a 1-1 and onto map $l\colon V_1 \to V_2$ satisfying
$\mathcal{T}_N(m) = \mathcal{T}_N(l(m)), \mathcal{I}_N(m) = \mathcal{I}_N((m))$ and $\mathcal{F}_N(m) = \mathcal{F}_M(l(m))$ for all $h \in V_1$
$\mathcal{T}_M(mk) = \mathcal{T}_M(l(mk)), \mathcal{I}_M(mk) = \mathcal{I}_M((mk))$ and $\mathcal{F}_M(mk) = \mathcal{F}_M(l(mk))$ for all $mn \in E_1$

$\overline{T}_M(mk) = \mathcal{T}_N(m) \wedge \mathcal{T}_N(k) - \mathcal{T}_M(mk)$

$= \mathcal{T}_N(l(m)) \wedge \mathcal{T}_N(l(k)) - \mathcal{T}_M(l(mk))$

$\overline{I}_M(mk) = \mathcal{I}_N(m) \wedge \mathcal{I}_N(k) - \mathcal{I}_M(mk)$

$= \mathcal{I}_N((m)) \wedge \mathcal{I}_N((k)) - \mathcal{I}_M((mk))$

$\overline{F}_M(mk) = \mathcal{F}_N(m) \wedge \mathcal{F}_N(k) - \mathcal{F}_M(mk)$

$= \mathcal{F}_N(l(m)) \wedge \mathcal{F}_N(l(k)) - \mathcal{F}_M(l(mk))$

Hence $\overline{G}_{R1} \cong \overline{G}_{R2}$. The converse part is similar.

Theorem 2.17. *Let $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ be the two highly streak erratic neutrosophic fuzzy graphs. If $\mathcal{G}_{R1}$ is weak isomorphism with $\mathcal{G}_{R2}$, then $\overline{G}_{R1}$ is weak isomorphic with $\overline{G}_{R2}$.*

Proof. If l is weak isomorphism between $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ then $l\colon V_1 \to V_2$ is a 1-1 and onto function such that,

$\mathcal{T}_N(m) = \mathcal{T}_N(l(m)), \mathcal{I}_N(m) = \mathcal{I}_N(l(m))$ and $\mathcal{F}_N(m) = \mathcal{F}_N(l(m))$

$\mathcal{T}_M(mk) \leq \mathcal{T}_M(l(m)l(k)), \mathcal{I}_M(mk) \leq \mathcal{I}_M((m)l(k))$ and $\mathcal{F}_M(mk) \leq \mathcal{F}_M(l(m)l(k))$

As, $l^{-1}\colon V_2 \to V_1$ is also 1-1, onto, for every $x_2 \in V_2$, there exists $x_1 \in V_1$ such that $k^{-1}(\mathcal{G}_{R2}) = x_1$

$\mathcal{T}_N(m_1) = \mathcal{T}_N(l^{-1}(m_2)), \mathcal{I}_N(m_1) = \mathcal{I}_N(^{-1}(m_2))$ and $\mathcal{F}_N(m_1) = \mathcal{F}_N(l^{-1}(m_2))$

$$\mathcal{T}_M(m_1k_1) = \mathcal{T}_N(m_1) \wedge \mathcal{T}_N(k_1) - \mathcal{T}_M(m_1k_1)$$

$$\overline{T}_M(l^{-1}(m_2)l^{-1}(k_2)) \geq \mathcal{T}_N(l(m_1)) \wedge \mathcal{T}_N(l(k_1)) - \mathcal{T}_M(l(m_1)l(k_1))$$

$$= \mathcal{T}_N(m_2) \wedge \mathcal{T}_N(k_2) - \mathcal{T}_M(m_2k_2) = \overline{T}_M(m_2k_2)$$

$$\overline{T}_M(m_2k_2) \leq \overline{T}_M(l^{-1}(m_2)k^{-1}(y_2))$$

$$\overline{I}_M(m_1k_1) = \mathcal{I}_N(m_1) \vee \mathcal{I}_N(k_1) - \mathcal{I}_M(m_1k_1)$$

$$\overline{I}_M(l^{-1}(m_2)l^{-1}(k_2)) \leq \mathcal{I}_N(l(m_1)) \vee \mathcal{I}_N(l(k_1)) - \mathcal{I}_M(l(m_1)l(k_1))$$

$$= \mathcal{I}_N(m_2) \vee \mathcal{I}_N(k_2) - \mathcal{I}_M(m_2k_2) = \overline{I}_M(m_2k_2)$$

$$\overline{I}_M(m_2k_2) \geq \overline{I}_M(l^{-1}(m_2)l^{-1}(k_2))$$

$$\overline{F}_M(m_1k_1) = \mathcal{F}_N(m_1) \vee \mathcal{F}_N(k_1) - \mathcal{F}_M(m_1k_1)$$

$$\overline{F}_M(l^{-1}(m_2)l^{-1}(k_2)) \leq \mathcal{F}_N(l(m_1)) \vee \mathcal{F}_N(l(k_1)) - \mathcal{F}_M(l(m_1)l(k_1))$$

$$= \mathcal{F}_N(m_2) \vee \mathcal{F}_N(k_2) - \mathcal{F}_M(m_2k_2) = \overline{F}_M(m_2k_2)$$

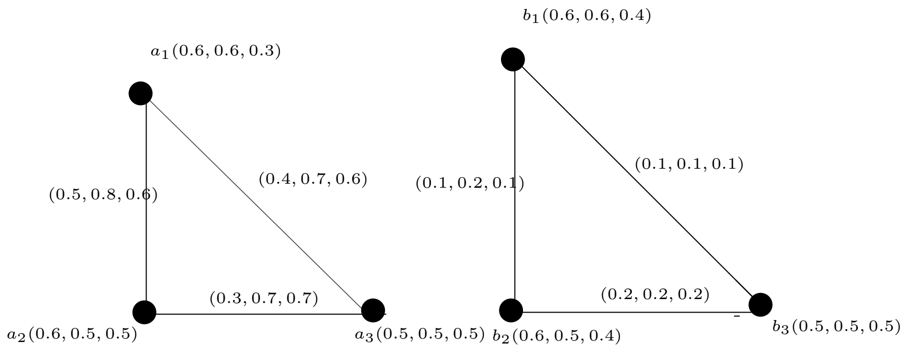
$$\overline{F}_M(m_2k_2) \geq \overline{F}_M(l^{-1}(m_2)l^{-1}(k_2))$$

So, $l^{-1}\colon V_1 \to V_2$ is weak isomorphism between $\overline{G}_{R1}$ and $\overline{G}_{R2}$.

Remark 2.18. *Theorem 2.17 is true for highly totally streak erratic neutrosophic fuzzy graphs.*

Remark 2.19. *A neighbourly streak erratic neutrosophic fuzzy graph need not be self complementary.*

Example 2.20. Consider the neutrosophic fuzzy graph on $H_R^*(V, E)$.



H $\bar{H}$

Remark 2.21. *Theorem 2.17 is true for highly totally streak erratic neutrosophic fuzzy graphs.*

Remark 2.22. *A neighbourly streak erratic neutrosophic fuzzy graph need not be self complementary.*

Theorem 2.23. *Isomorphism between the neighbourly streak erratic neutrosophic fuzzy graphs has an equivalent relation.*

Proof. Let $\mathcal{G}_{R1} = (\mathcal{T}_1, \mathcal{I}_1, \mathcal{F}_1)$, $\mathcal{G}_{R2} = (\mathcal{T}_2, \mathcal{I}_2, \mathcal{F}_2)$ and $\mathcal{G}_{R3} = (\mathcal{T}_3, \mathcal{I}_3, \mathcal{F}_3)$ be neighbourly streak erratic neutrosophic fuzzy graphs with vertex sets V_1, V_2 and V_3 respectively. Reflexive: (i.e) To prove $\mathcal{G} \cong \mathcal{G}$. Imagine that the identity map $l\colon V \to V$ such that $l(u) = u$ for all $u \in V$. Clearly l is a 1-1, onto map satisfying $\mathcal{T}_{N1}(u) = \mathcal{T}_{N1}(k(u))$, $\mathcal{I}_{N1}(u) = \mathcal{I}_{N1}(k(u))$, $\mathcal{F}_{N1}(u) = \mathcal{F}_{N1}(k(u))$ and $\mathcal{T}_{M1}(uv) = \mathcal{T}_{M1}((k(u)k(v))$,$\mathcal{I}_{M1}(uv) = \mathcal{I}_{M1}((k(u)k(v))$, $\mathcal{F}_{M1}(uv) = \mathcal{F}_{M1}((k(u)k(v))$ for all $u, v \in V$.

Therefore l is an isomorphism of the neighbourly streak erratic neutrosophic fuzzy graph to itself. Hence l satisfies reflexive relation.

Symmetric: To prove, if $\mathcal{G}_{R1} \cong \mathcal{G}_{R2}$ then $\mathcal{G}_{R2} \cong \mathcal{G}_{R1}$.

Assume $\mathcal{G}_{R1} \cong \mathcal{G}_{R2}$. Let $l\colon V_1 \to V_2$ be an isomorphism from $\mathcal{G}_{R1}$ onto $\mathcal{G}_{R2}$ such that $l(u_1) = u_2$, for all $u_1 \in V_1$. This l is a 1-1, onto map satisfying

$\mathcal{T}_{N1}(u_1) = \mathcal{T}_{N2}(l(u_1))$, $\mathcal{I}_{N1}(u_1) = \mathcal{I}_{N2}(l(u_1))$ and $\mathcal{F}_{N1}(u_1) = \mathcal{F}_{N2}(l(u_1))$

$\mathcal{T}_{M1}(u_1v_1) = \mathcal{T}_{M2}(l(u_1)l(v_1))$,$\mathcal{I}_{M1}(u_1v_1) = \mathcal{I}_{M2}(l(u_1)l(v_1))$ and $\mathcal{F}_{M1}(u_1v_1) = \mathcal{F}_{M2}(l(u_1)l(v_1))$

Since l is 1-1, onto, inverse exists. So, $l^{-1}(u_2) = u_1$, for all $u_2 \in V_2$.

$\mathcal{T}_{N1}(l^{-1}(u_2)) = \mathcal{T}_{N2}(u_1)$,$\mathcal{I}_{N1}(l^{-1}(u_2)) = \mathcal{I}_{N2}(u_1)$ and $\mathcal{F}_{N1}(l^{-1}(u_2)) = \mathcal{F}_{N2}(u_1)$ for all $u_2 \in V_2$ and

$\mathcal{T}_{N1}(l^{-1}(u_2)l^{-1}(v_2)) = \mathcal{T}_{N2}(u_1v_1)$,$\mathcal{I}_{N1}(l^{-1}(u_2)l^{-1}(v_2)) = \mathcal{I}_{N2}(u_1v_1)$ and $\mathcal{F}_{N1}(l^{-1}$ (u_2) l^{-1} $(v_2)) = \mathcal{F}_{N2}(u_1v_1)$ for all $u_2, v_2 \in V_2$.

Thus, $l^{-1}\colon V_2 \to V_1$ is a 1-1, onto map which is an isomorphism from $\mathcal{G}_{R2}$ to $\mathcal{G}_{R1}$.

Transitive: To Prove: if $\mathcal{G}_{R1} \cong \mathcal{G}_{R2}$ and $\mathcal{G}_{R2} \cong \mathcal{G}_{R3}$ then $\mathcal{G}_{R1} \cong \mathcal{G}_{R3}$

Assume $\mathcal{G}_{R1} \cong \mathcal{G}_{R2}$. Let $l\colon V_1 \to V_2$ be an isomorphism of $\mathcal{G}_{R1}$ onto $\mathcal{G}_{R2}$ such that $l(u_1) = u_2$ for all $u_1 \in V_1$ satisfying $\mathcal{T}_{N1}(u_1) = \mathcal{T}_{N2}(k(u_1)) = \mathcal{T}_{N2}(u_2)$, $\mathcal{I}_{N1}(u_1) = \mathcal{I}_{N2}(k(u_1)) = \mathcal{I}_{N2}(u_2)$, $\mathcal{F}_{N1}(u_1) = \mathcal{F}_{N2}(k(u_1)) = \mathcal{F}_{N2}(u_2)$ for all $u_1 \in V_1$.

$\mathcal{T}_{M1}(u_1v_1) = \mathcal{T}_{M2}(l(u_1)l(v_1)) = \mathcal{T}_{M2}(u_2v_2)$, $\mathcal{I}_{M1}(u_1v_1) = \mathcal{I}_{M2}(l(u_1)l(v_1)) = \mathcal{I}_{M2}(u_2v_2)$ $\mathcal{F}_{M1}(u_1v_1) = \mathcal{F}_{M2}(l(u_1)l(v_1)) = \mathcal{F}_{M2}(u_2v_2)$ for all $u_1, v_1 \in V_1$.

Assume $\mathcal{G}_{R2} \cong \mathcal{G}_{R3}$. Let $k\colon V_2 \to V_3$ be an isomorphism of $\mathcal{G}_{R2}$ onto $\mathcal{G}_{R3}$ such that $k(u_2) = u_3$ for all $u_2 \in V_2$ satisfying $\mathcal{T}_{N2}(u_2) = \mathcal{T}_{N3}(k(u_2)) = \mathcal{T}_{N3}(u_3)$, $\mathcal{I}_{N2}(u_2) = \mathcal{I}_{N3}(k(u_2)) = \mathcal{I}_{N3}(u_3)$,$\mathcal{F}_{N2}(u_2) = \mathcal{F}_{N3}(k(u_2)) = \mathcal{F}_{N3}(u_3)$ for all $u_2 \in V_2$

$\mathcal{T}_{N2}(u_2v_2) = \mathcal{T}_{N3}(k(u_2)k(v_2)) = \mathcal{T}_{N3}(u_3v_3)$,$\mathcal{I}_{N2}(u_2v_2) = \mathcal{I}_{N3}(k(u_2)k(v_2)) = \mathcal{I}_{N3}(u_3v_3)$, $\mathcal{F}_{N2}(u_2v_2) = \mathcal{F}_{N3}(k(u_2)k(v_2)) = \mathcal{F}_{N3}(u_3v_3)$ for all $u_2, v_2 \in V_2$

Since $l\colon V_1 \to V_2$ and $k\colon V_2 \to V_3$ are isomorphism from $\mathcal{G}_{R1}$ onto $\mathcal{G}_{R2}$ and $\mathcal{G}_{R2}$ onto $\mathcal{G}_{R3}$, then $k \circ l$ is 1-1, onto map from V_1 to V_3. So, $k \circ l\colon V_1 \to V_3$ where $(k \circ l)(u) = k(l(u))$

Now, $\mathcal{T}_{N1}(u_1) = \mathcal{T}_{N2}(l(u_1)) = \mathcal{T}_{N2}(u_2) = \mathcal{T}_{N3}(k(u_2)) = \mathcal{T}_{N3}(k(l(u_1)))$

$\mathcal{I}_{N1}(u_1) = \mathcal{I}_{N2}(l(u_1)) = \mathcal{I}_{N2}(u_2) = \mathcal{I}_{N3}(k(u_2)) = \mathcal{I}_{N3}(k(l(u_1)))$

$\mathcal{F}_{N1}(u_1) = \mathcal{F}_{N2}(l(u_1)) = \mathcal{F}_{N2}(u_2) = \mathcal{F}_{N3}(k(u_2)) = \mathcal{F}_{N3}(k(l(u_1)))$

$$\begin{aligned}\mathcal{T}_{N1}(u_1v_1) &= \mathcal{T}_{N2}(k(u_1)l(v_1)) = \mathcal{T}_{N2}(u_2v_2) = \mathcal{T}_{N3}(k(u_2)k(v_2))\\ &= \mathcal{T}_{N3}(k(l(u_1)), k(l(v_1)))\end{aligned}$$

$$\begin{aligned}\mathcal{I}_{N1}(u_1v_1) &= \mathcal{I}_{N2}(k(u_1)l(v_1)) = \mathcal{I}_{N2}(u_2v_2) = \mathcal{I}_{N3}(k(u_2)k(v_2))\\ &= \mathcal{I}_{N3}(k(l(u_1)), k(l(v_1)))\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{N1}(u_1v_1) &= \mathcal{F}_{N2}(k(u_1)l(v_1)) = \mathcal{F}_{N2}(u_2v_2) = \mathcal{F}_{N3}(k(u_2)k(v_2))\\ &= \mathcal{F}_{N3}(k(l(u_1)), k(l(v_1)))\end{aligned}$$

Thus, $k \circ l$ is an isomorphism between $\mathcal{G}_{R1}$ and $\mathcal{G}_{R3}$.

Hence the isomorphism between the neighbourly streak erratic neutrosophic fuzzy graphs is an equivalent relation.

3. Isomorphic Properties of μ-Complement of Highly Streak erratic Neutrosophic Fuzzy Graphs

Definition 3.1. *Assume that $H : (\mathcal{N}, \mathcal{M})$ is a neutrosophic fuzzy graph where the μ-complement of H is defined as $H^{\mu}_{R} : (A, B^{\mu})$, $B^{\mu} = (\mathcal{T}^{\mu}_{M}, \mathcal{I}^{\mu}_{M}, \mathcal{F}^{\mu}_{M})$*

$$\mathcal{T}^{\mu}_{M}(mk) = \begin{cases} \mathcal{T}_N(m) \wedge \mathcal{T}_N(k) - \mathcal{T}_M(mk) & \mathcal{T}_M(mk) > 0 \\ 0 & \mathcal{T}_M(mk) = 0 \end{cases}$$

$$\mathcal{I}^{\mu}_{M}(mk) = \begin{cases} \mathcal{I}_M(mk) - \mathcal{I}_N(m) \vee \mathcal{I}_N(k) & \mathcal{I}_M(mk) > 0 \\ 0 & \mathcal{I}_M(mk) = 0 \end{cases}$$

$$\mathcal{F}^{\mu}_{M}(mk) = \begin{cases} \mathcal{F}_M(mk) - \mathcal{F}_N(m) \vee \mathcal{F}_N(k) & \mathcal{F}_M(mk) > 0 \\ 0 & \mathcal{F}_M(mk) = 0 \end{cases}$$

Remark 3.2. *Let the μ-complement of highly erratic neutrosophic fuzzy graph need not be highly streak erratic.*

Theorem 3.3. *Consider that the $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are the two highly streak erratic neutrosophic fuzzy graphs. If $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are isomorphic then μ-complement of $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are similarly isomorphic but the complements may not be highly streak erratic.*

Proof. Assume that $\mathcal{G}_{R1}$ and $\mathcal{G}_{R2}$ are isomorphic where 1-1 and onto map $l\colon V_1 \to V_2$ be suitable for

$\mathcal{T}_N(m) = \mathcal{T}_N(l(m))$, $\mathcal{I}_N(m) = \mathcal{I}_N(l(m))$, $\mathcal{F}_N(m) = \mathcal{F}_N(l(m))$

$\mathcal{T}_M(mk) = \mathcal{T}_M(l(m)l(k))$, $\mathcal{I}_M(mk) = \mathcal{I}_M(l(m)l(k))$, $\mathcal{F}_M(mk) = \mathcal{F}_M(l(m)l(k))$

By definition of μ-complement,

$\mathcal{T}^{\mu}_M(mk) = \mathcal{T}_N(m) \wedge \mathcal{T}_N(k) - \mathcal{T}_M(mk) = \mathcal{T}_M(l(m)) \wedge \mathcal{T}_N(l(k)) - \mathcal{T}_M(l(m)l(k))$

$\mathcal{I}^{\mu}_M(mk) = \mathcal{I}_N(m) \wedge \mathcal{I}_N(k) - \mathcal{I}_M(mk) = \mathcal{I}_M(l(m)) \wedge \mathcal{I}_N(l(k)) - \mathcal{I}_M(l(m)l(k))$

$\mathcal{F}^{\mu}_M(mk) = \mathcal{F}_N(m) \wedge \mathcal{F}_N(k) - \mathcal{F}_M(mk) = \mathcal{F}_M(l(m)) \wedge \mathcal{F}_N(l(k)) - \mathcal{F}_M(l(m)l(k))$

Hence $\bar{\mathcal{G}}_{R1} \sim \bar{\mathcal{G}}_{R2}$.

Definition 3.4. *A neutrosophic fuzzy graph H is said to be self μ-complementary if $H \sim H^{\mu}_R$.*

Theorem 3.5. *Let H be self μ-complementary highly streak erratic neutrosophic fuzzy graph, then*

$\sum \mathcal{T}_M(uv) = \frac{1}{2}\sum \mathcal{T}_N(u) \wedge \mathcal{T}_N(v)$, $\sum \mathcal{I}_M(uv) = \frac{1}{2}\sum \mathcal{I}_N(u) \vee \mathcal{I}_N(v)$ *and* $\sum \mathcal{F}_M(uv) = \frac{1}{2}\sum \mathcal{F}_N(u) \vee \mathcal{F}_N(v)$.

Proof. Let H be self μ-complementary highly streak erratic neutrosophic fuzzy graph. Since $H \sim H^{\mu}$, there exists a 1-1 and onto map $l\colon V \to V$ such that

$\mathcal{T}_N(u) = \mathcal{T}^{\mu}_N(l(u)) = \mathcal{T}_N(l(u))$, $\mathcal{I}_N(u) = \mathcal{I}^{\mu}_N(l(u)) = \mathcal{I}_N(l(u))$ and $\mathcal{I}_N(u) = \mathcal{I}^{\mu}_N(l(u)) = \mathcal{I}_N(l(u))$

$\mathcal{T}_M(uv) = \mathcal{T}^{\mu}_M(l(u)l(v))$, $\mathcal{I}_M(uv) = \mathcal{I}^{\mu}_M(l(u)l(v))$ and $\mathcal{F}_M(uv) = \mathcal{F}^{\mu}_M(l(u)l(v))$

By definition of -complement, we have

$$\mathcal{T}^{\mu}_M(mk) = \begin{cases} \mathcal{T}_N(m) \wedge \mathcal{T}_N(k) - \mathcal{T}_M(mk) & \mathcal{T}_M(mk) > 0 \\ 0 & \mathcal{T}_M(mk) = 0 \end{cases}$$

$$\mathcal{I}^{\mu}_M(mk) = \begin{cases} \mathcal{I}_N(m) \wedge \mathcal{I}_N(k) - \mathcal{I}_M(mk) & \mathcal{I}_M(mk) > 0 \\ 0 & \mathcal{I}_M(mk) = 0 \end{cases}$$

$$\mathcal{F}^{\mu}_M(mk) = \begin{cases} \mathcal{F}_N(m) \wedge \mathcal{F}_N(k) - \mathcal{F}_M(mk) & \mathcal{F}_M(mk) > 0 \\ 0 & \mathcal{F}_M(mk) = 0 \end{cases}$$

Now, $\mathcal{T}_M(uv) = \mathcal{T}_N(l(u)) \wedge \mathcal{T}_N(l(v)) - \mathcal{T}_M(l(u)l(v))$

$\mathcal{T}_M(uv) + \mathcal{T}_M(l(u)l(v)) = \mathcal{T}_N(l(u)) \wedge \mathcal{T}_N(l(v)) \Rightarrow 2\mathcal{T}_M(uv) = \mathcal{T}_N(u) \wedge \mathcal{T}_N(v)$

$\Rightarrow 2\sum \mathcal{T}_M(uv) = \sum \mathcal{T}_N(u) \wedge \mathcal{T}_N(v) \Rightarrow \sum \mathcal{T}_M(uv) = \frac{1}{2}\sum \mathcal{T}_N(u) \wedge \mathcal{T}_N(v)$

Similarly, we can show that $\sum \mathcal{I}_M(uv) = \frac{1}{2}\sum \mathcal{I}_N(u) \vee \mathcal{I}_N(v)$ and $\sum \mathcal{F}_M(uv) = \frac{1}{2}\sum \mathcal{F}_N(u) \vee \mathcal{F}_N(v)$.

Remark 3.6. *Even though G is highly streak erratic neutrosophic fuzzy graph with $\sum \mathcal{T}_M(uv) = \frac{1}{2}\sum \mathcal{T}_N(u) \wedge \mathcal{T}_N(v)$, $\sum \mathcal{I}_M(uv) = \frac{1}{2}\sum \mathcal{I}_N(u) \vee \mathcal{I}_N(v)$ and $\sum \mathcal{F}_M(uv) = \frac{1}{2}\sum \mathcal{F}_N(u) \vee \mathcal{F}_N(v)$ then G need not be self μ-complementary.*

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